# Primitive Rules of Inference

### Assumption — A

| | | | |
|---|---|---|---|
| $n$ | $(n)$ | $\Phi$ | A |

*$\Phi$ can be any wff whatever.*

*This space intentionally left void of any useful information*

### Ampersand-Elimination — &E

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(m)$ | $(\Phi \& \Psi)$ | ... |
| $i_1,...i_x$ | $(n)$ | $\Phi$ | $m$ &E |

*Can also conclude $\Psi$.*

### Ampersand-Introduction — &I

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(k)$ | $\Phi$ | ... |
| $j_1,...j_y$ | $(m)$ | $\Psi$ | ... |
| $i_1,...i_x,j_1,...j_y$ | $(n)$ | $(\Phi \& \Psi)$ | $k,m$ &I |

### Wedge-Elimination — ∨E

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(k)$ | $(\Phi \vee \Psi)$ | ... |
| $j_1,...j_x$ | $(m)$ | $\sim\Phi$ | ... |
| $i_1,...i_x,j_1,...j_y$ | $(n)$ | $\Psi$ | $k,m$ ∨E |

*If line $m$ is $\sim\Psi$, conclude $\Phi$.*
*Also if line $k = (\sim\Phi \vee \Psi)$, line $m = \Phi$, etc.*

### Wedge-Introduction — ∨I

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(m)$ | $\Phi$ | ... |
| $i_1,...i_x$ | $(n)$ | $(\Phi \vee \Psi)$ | $m$ ∨I |

*Can also conclude $(\Psi \vee \Phi)$.*

### Double-Arrow-Elimination — ↔E

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(m)$ | $(\Phi \leftrightarrow \Psi)$ | ... |
| $i_1,...i_x$ | $(n)$ | $(\Phi \rightarrow \Psi)$ | $m$ ↔E |

*Can also conclude $(\Psi \rightarrow \Phi)$.*

### Double-Arrow-Introduction — ↔I

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(m)$ | $(\Phi \rightarrow \Psi)$ | ... |
| $j_1,...j_y$ | $(n)$ | $(\Psi \rightarrow \Phi)$ | ... |
| $i_1,...i_x,j_1,...j_y$ | $(n)$ | $(\Phi \leftrightarrow \Psi)$ | $m,n$ ↔I |

*Order does not matter: can also conclude $(\Psi \leftrightarrow \Phi)$.*

### Arrow-Elimination — →E

| | | | |
|---|---|---|---|
| $i_1,...i_x$ | $(k)$ | $(\Phi \rightarrow \Psi)$ | ... |
| $j_1,...j_x$ | $(m)$ | $\Phi$ | ... |
| $i_1,...i_x,j_1,...j_y$ | $(n)$ | $\Psi$ | $k,m$ →E |

*You CANNOT conclude $\Phi$ if line $m = \Psi$.*

### Arrow-Introduction — →I

| | | | |
|---|---|---|---|
| $k$ | $(k)$ | $\Phi$ | A |
| $i_1,...i_x,k$ | $(m)$ | $\Psi$ | ... |
| $i_1,...i_x$ | $(n)$ | $(\Phi \rightarrow \Psi)$ | $m$ →I($k$) |

*Line $k$ MUST be an assumption.*
*$k$ is dropped from the assumption set for line $n$.*
*$k$ does not have to be in the assumption set for line $m$, but in practice it usually is.*

*In all rules, there may be other lines between the lines indicated.*

*Assumption sets are indicated by strings like $i_1,...i_x$ for convenience. In an actual proof, this will be a string of actual numbers, e.g. $1,2,4,7$. $i_1,...i_x,j_1,...j_y$ means "the string of numbers that includes everything in $i_1,...i_x$ and $j_1,...j_y$"*

### Reductio ad Absurdum — RAA

| | | | |
|---|---|---|---|
| $k$ | $(k)$ | $\Phi$ | A |
| $i_1,...i_x,k$ | $(l)$ | $\Psi$ | ... |
| $j_1,...j_y,k$ | $(m)$ | $\sim\Psi$ | ... |
| $i_1,...i_x,j_1,...j_y$ | $(n)$ | $\sim\Phi$ | $l,m$ RAA($k$) |

*Line $k$ MUST be an assumption.*
*Can also assume $\sim\Phi$ and conclude $\Phi$.*
*$k$ is dropped from the assumption set for line $n$.*
*$k$ does not have to be in the assumption set for line $l$ or line $m$, but in practice it usually is.*

# Strategies for Proofs

## Working Backwards

Take the conclusion as your goal and then use an appropriate strategy to see what you need in order to get there. This will give you a new goal. Apply a strategy to that goal in turn, and continue until what you need is something that you already have. You can then reverse direction and complete the proof.

## Taking things apart (Elimination rules)

Use one of these strategies when you see the wff that you are trying to get is already present as a constituent of a line that you have already added. They tell you how to get it out of that line and what else you need in order to do that (if it's possible).

| If your goal is ... | and you have ... | then the rule to use is ... | and you also need ... |
|---|---|---|---|
| $\Phi$ | $\Phi \& \Psi$ or $\Psi \& \Phi$ | &E | nothing else |
| $\Phi$ | $\Phi \vee \Psi$ | $\vee$E | $\sim\Psi$ |
| $\Phi \rightarrow \Psi$ | $\Phi \leftrightarrow \Psi$ or $\Psi \leftrightarrow \Phi$ | $\leftrightarrow$E | nothing else |
| $\Phi$ | $\Psi \rightarrow \Phi$ | $\rightarrow$E | $\Psi$ |
| $\Phi$ | $\Phi \rightarrow \Psi$ | another strategy | help |

## Building things up (Introduction rules)

Try one of these strategies if the wff that you are trying to get is not already present as a constituent of another line you have already added. They tell you how to build it up and what else you need in order to do that.

| If your goal is ... | then the rule to use is ... | and you need ... | and ... |
|---|---|---|---|
| $\Phi \& \Psi$ | &I | $\Phi$ | $\Psi$ |
| $\Phi \vee \Psi$ | $\vee$I | $\Phi$ or $\Psi$ | (nothing else) |
| $\Phi \leftrightarrow \Psi$ | $\leftrightarrow$I | $\Phi \rightarrow \Psi$ | $\Psi \rightarrow \Phi$ |
| $\Phi \rightarrow \Psi$ | $\rightarrow$I | to assume $\Phi$ | to deduce $\Psi$ |
| $\sim\Phi$ | RAA | to assume $\Phi$ | to deduce $\Psi$ and $\sim\Psi$ |